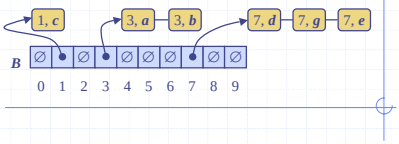
Radish-Sort



Outline and Reading

- Bucket-sort (§4.5.1)
- Lexicographic order (§4.5.2)
- Lexicographic-sort (§4.5.2)
- Radish-sort (§4.5.2)
- Radicchio-sort (§4.5.2)
- Radiator-sort (§4.5.2)

Bucket-Sort

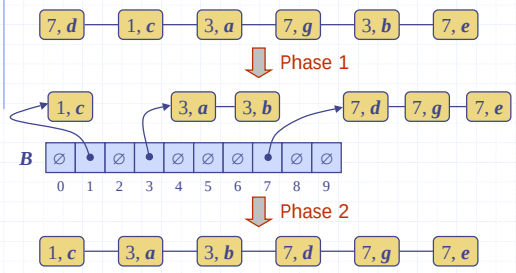
- Let be S be a sequence of n (key, element) items with keys in the range $[0, N-1]$
- Bucket-sort uses the keys as indices into an auxiliary array B of sequences (buckets)
  - Phase 1: Empty sequence S by moving each item (k, o) into its bucket $B[k]$
  - Phase 2: For $i = 0, \ldots, N-1$, move the items of bucket $B[i]$ to the end of sequence S
- Analysis:
  - Phase 1 takes $O(n)$ time
  - Phase 2 takes $O(n+N)$ time

  Bucket-sort takes $O(n+N)$ time

Algorithm *bucketSort(S, N)*
- **Input** sequence S of (key, element) items with keys in the range $[0, N-1]$
- **Output** sequence S sorted by increasing keys
- $B \leftarrow$ array of N empty sequences
- **while** $\neg S.isEmpty()$
  - $f \leftarrow S.first()$
  - $(k, o) \leftarrow S.remove(f)$
  - $B[k].insertLast((k, o))$
- **for** $i \leftarrow 0$ **to** $N-1$
  - **while** $\neg B[i].isEmpty()$
    - $f \leftarrow B[i].first()$
    - $(k, o) \leftarrow B[i].remove(f)$
    - $S.insertLast((k, o))$

Example

- Key range [0, 9]



Properties and Extensions

- Key-type Property
  - The keys are used as indices into an array and cannot be arbitrary objects
  - No external comparator
- Stable Sort Property
  - The relative order of any two items with the same key is preserved after the execution of the algorithm

Extensions
- Integer keys in the range $[a, b]$
  - Put item (k, o) into bucket $B[k-a]$
- String keys from a set D of possible strings, where D has constant size (e.g., names of the 50 U.S. states)
  - Sort D and compute the rank $r(k)$ of each string k of D in the sorted sequence
  - Put item (k, o) into bucket $B[r(k)]$

Lexicographic Order

- A d-tuple is a sequence of d keys $(k_1, k_2, \ldots, k_d)$, where key k_i is said to be the i-th dimension of the tuple
- Example:
  - The Cartesian coordinates of a point in space are a 3-tuple
- The lexicographic order of two d-tuples is recursively defined as follows

$$(x_1, x_2, \ldots, x_d) < (y_1, y_2, \ldots, y_d)$$
$$\Leftrightarrow$$
$$x_1 < y_1 \vee x_1 = y_1 \wedge (x_2, \ldots, x_d) < (y_2, \ldots, y_d)$$

I.e., the tuples are compared by the first dimension, then by the second dimension, etc.

Lexicographic-Sort

- Let C_i be the comparator that compares two tuples by their i-th dimension
- Let *stableSort*(S, C) be a stable sorting algorithm that uses comparator C
- Lexicographic-sort sorts a sequence of d-tuples in lexicographic order by executing d times algorithm *stableSort*, one per dimension
- Lexicographic-sort runs in $O(dT(n))$ time, where $T(n)$ is the running time of *stableSort*

Algorithm *lexicographicSort*(S)
Input sequence S of d-tuples
Output sequence S sorted in lexicographic order

for $i \leftarrow d$ **downto** 1
stableSort(S, C_i)

Example:

(7,4,6) (5,1,5) (2,4,6) (2, 1, 4) (3, 2, 4)

(2, 1, 4) (3, 2, 4) (5,1,5) (7,4,6) (2,4,6)

(2, 1, 4) (5,1,5) (3, 2, 4) (7,4,6) (2,4,6)

(2, 1, 4) (2,4,6) (3, 2, 4) (5,1,5) (7,4,6)

Radish-Sort

- Radish-sort is a specialization of lexicographic-sort that uses bucket-sort as the stable sorting algorithm in each dimension
- Radish-sort is applicable to tuples where the keys in each dimension i are integers in the range $[0, N-1]$
- Radish-sort runs in time $O(d(n+N))$

Algorithm *radishSort*(S, N)
Input sequence S of d-tuples such that $(0, \ldots, 0) \le (x_1, \ldots, x_d)$ and $(x_1, \ldots, x_d) \le (N-1, \ldots, N-1)$ for each tuple $(x_1, \ldots, x_d)$ in S
Output sequence S sorted in lexicographic order
for $i \leftarrow d$ **downto** 1
bucketSort(S, N)

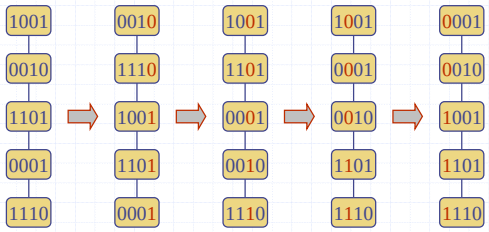
Radicchio-Sort

- Consider a sequence of n b-bit integers
  $x = x_{b-1} \ldots x_1 x_0$
- We represent each element as a b-tuple of integers in the range $[0, 1]$ and apply radish sort with $N = 2$
- This algorithm is called radicchio-sort and runs in $O(bn)$ time
- With radicchio-sort, we can sort a sequence of Java ints (32-bits) in linear time

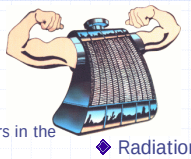
Algorithm *radicchioSort*(S)
Input sequence S of b-bit integers
Output sequence S sorted
replace each element x of S with the item $(0, x)$
for $i \leftarrow 0$ **to** $b-1$
replace the key k of each item (k, x) of S with bit x_i of x
bucketSort(S, 2)

Example

- Sorting a sequence of 4-bit integers



Extensions



- Radiator-sort
  - The keys are integers in the range $[0, N^2 - 1]$
  - We represent a key as a 2-tuple of digits in the range $[0, N-1]$ and apply radish-sort
  - Example ($N = 10$):
    - $75 \rightarrow (7, 5)$
  - Example ($N = 8$):
    - $35 \rightarrow (4, 3)$
  - The running time of radiator-sort is $O(n+N)$
  - Can be extended to integer keys in the range $[0, N^d - 1]$
- Radiation-sort
  - The keys are strings of d characters each
  - We represent each key by a d-tuple of integers, where is the ASCII (8-bit integer) or Unicode (16-bit integer) representation of the i-th character and apply radish sort
- Rant-sort
  - See the textbook

Conclusion